WestVirginiaUniversity
DEPARTMENT OF MATHEMATICS

WVU Algebra Seminar via Zoom

Test modules, weakly regular homomorphisms, and complete intersection dimension

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Abstract: Let (R, m_R, K_R) be a local (Noetherian) ring.

A finitely generated R-module (resp. homologically finite complex) T is said to be a (pd-)test module (resp. a (pd-)test complex) if the following condition holds for any finitely generated R-module M (respectively for any homologically finite complex): $\mathrm{Tor}_i^R(M,T) = 0$ for $i \gg 0$ if and only if $\mathrm{pd_R(M)} < \infty$.

For instance, it is a basic result in commutative algebra that K_R is a test module (and a test complex) for R. On the other hand, in **[AGP97]**, Avramov, Gasharov, and Peeva showed that R is a complete intersection ring provided K_R has finite complete intersection dimension.

This talk is a report on my recent work where I answered affirmatively the following question proposed by O. Celikbas, H. Dao, and R. Takahashi:

Question 1: (see **[CDT14]**) Let R be a local ring. Let M be a test module with CI-$\mathrm{dim_R}(M) < \infty$. Then must R be a complete intersection?

In order to answer the above question, I studied the following question due to O. Celikbas and S.S. Wagstaff and I answered it affirmatively in the case where K_R is uncountable, or the test complex T has finite complete intersection dimension:

Question 2: (see **[CW16]**) Let $\varphi: R \rightarrow (S, m_S)$ be a flat local homomorphism such that $S/m_R S$ is regular (in other words, φ is weakly regular homomorphism). If T is a test complex for R, then must $T \otimes_R S \ (= T \otimes_R^L S)$ be a test complex for S?

Bibliography

[AGP97] Avramov, L., Gasharov, V. N., & Peeva, V., *Complete intersection dimension*, Publ. Math., Inst. Hautes Etud. Sci. *,* **86**, (1997), 67-114.
[CDT14] Celikbas, O., Dao, H., & Takahashi, R., *Modules that detect finite homological dimensions,* Kyoto J. Math.*,* **54**(2), (2014), 295-310.
[CW16] Celikbas, O., S. Sather-Wagstaff, *Testing for the Gorenstein property*, Collect. Math., **67**(3), (2016), 555-568.